

(b.) Behaviour at the Origin

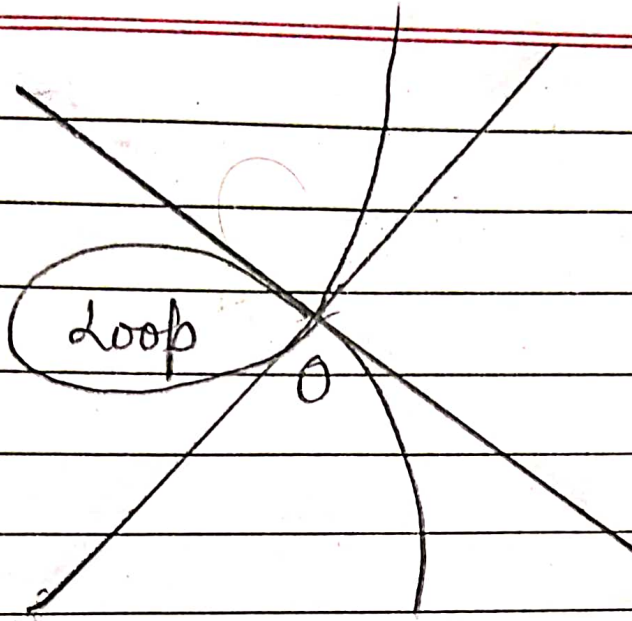
Observe whether the curve passes through the origin or not.

If it does, find the equation of the tangents to the curve at the origin in case of a closed curve. This means that the two branches of the curve cross each other, as it has been shown in the adjoining figure.

For example, let us consider the Folium of Descartes

$$x^2 + y^2 = 3axy.$$

Obviously it passes through the origin. To get the tangents to the curve at the origin equate the lowest degree terms to zero.



$$\therefore 3axy = 0.$$

$$\therefore x=0 \text{ and } y=0 \text{ as } 3a \neq 0$$

Thus $x=0$ and $y=0$ are two distinct tangents

(ii.) Let the tangents be real and coincident. In this case the origin is either a cusp (figures 1 and 2) or a double cusp (figure 3)

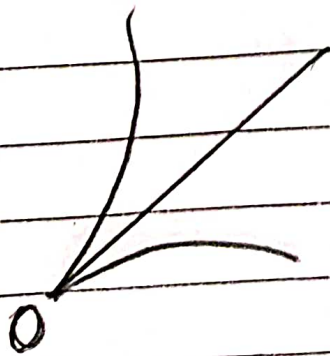


Fig 1

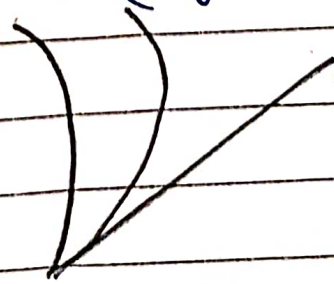


Fig 2

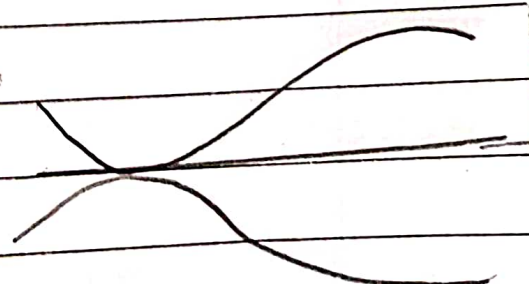


Fig 3.